Three Dimensional Geometry

Question1

If a line makes angles $90^\circ, 60^\circ$ and θ with x, y and z axes respectively, where θ is acute, then the value of θ is

KCET 2025

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: A

Solution:

A line's direction cosines along the x, y, z axes are

$\cos\alpha,\ \cos\beta,\ \cos\gamma$

where $\alpha = 90^\circ$, $\beta = 60^\circ$, and $\gamma = \theta$. They satisfy

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1\,.$

Compute the known cosines:

• $\cos\alpha = \cos 90^\circ = 0$

• $\cos\beta = \cos 60^\circ = \frac{1}{2}$

Plug in and solve for $\cos\gamma$:

$$0^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1 \quad\Longrightarrow\quad \frac{1}{4} + \cos^2\theta = 1 \quad\Longrightarrow\quad \cos^2\theta = \frac{3}{4}$$

Since θ is acute, $\cos\theta > 0$, so

$\cos\theta = \frac{\sqrt{3}}{2}\,.$

Therefore

$\theta = \arccos\!\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}\,.$

Answer: $\frac{\pi}{6}$.

Question2

The equation of the line through the point $(0, 1, 2)$ and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is

KCET 2025

Options:

A. $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

B. $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

C. $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

D. $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

Answer: B

Solution:

The given line has direction vector

$\mathbf{v} = (2, 3, -2)$.

We seek a direction $\mathbf{d} = (d_1, d_2, d_3)$ with

$\mathbf{v} \cdot \mathbf{d} = 2d_1 + 3d_2 - 2d_3 = 0.$

One convenient choice is

$d_1 = -3, \quad d_2 = 4, \quad d_3 = 3$

since

$2(-3) + 3(4) - 2(3) = -6 + 12 - 6 = 0.$

Hence the required line through $(0, 1, 2)$ is

$\frac{x-0}{-3} = \frac{y-1}{4} = \frac{z-2}{3},$

which is Option B.

Question3

The distance of the point $\mathrm{P}(-3, 4, 5)$ from yz plane is

KCET 2025

Options:

A. 4 units

B. 5 units

C. -3 units

D. 3 units

Answer: D

Solution:

The yz-plane is given by $x = 0$. So the distance from $P(x_0, y_0, z_0)$ to that plane is

$d = |x_0| = |-3| = 3.$

Answer: 3 units (Option D).

Question4

If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then k is equal to

KCET 2024

Options:

A. $-\frac{10}{7}$

B. $-\frac{7}{10}$

C. -10

D. -7

Answer: A

Solution:

$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$

$\therefore \quad \mathbf{b}_1 = -3\hat{\mathbf{i}} + 2k\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

and $\quad \mathbf{b}_2 = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

Now, $\quad \mathbf{b}_1 \cdot \mathbf{b}_2 = \mathbf{0}$

$$\Rightarrow -9k + 2k - 10 = 0$$
$$\Rightarrow -7k = 10$$
$$\therefore \quad k = \frac{-10}{7}$$

Hence, the value of k is $-\frac{10}{7}$.

Question5

The distance between the two planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is

KCET 2024

Options:

A. 2 units

B. 8 units

C. $\frac{2}{\sqrt{29}}$ unit

D. 4 units

Answer: C

Solution:

The equations of planes are

$2x + 3y + 4z = 4 \quad \text{.... (i)}$

$4x + 6y + 8z = 12$

$2x + 3y + 4z = 6 \quad \text{.... (ii)}$

It can be seen that the given planes are parallel. It is known that the distance between two parallel planes is $\left|\frac{d_2 - d_1}{\sqrt{a^2+b^2+c^2}}\right|$.

So, $\quad D = \left|\frac{6-4}{\sqrt{(2)^2+(3)^2+(4)^2}}\right| = \frac{2}{\sqrt{29}}$ units

----

Question6

The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{4-z}{-5}$ are the plane $2x - 2y + z = 5$ is

KCET 2024

Options:

A. $\frac{1}{5\sqrt{2}}$

B. $\frac{2}{5\sqrt{2}}$

C. $\frac{3}{50}$

D. $\frac{3}{\sqrt{50}}$

Answer: A

Solution:

We have, the equation of line as

$$\frac{x-2}{3}=\frac{y-3}{4}=\frac{4-z}{-5}$$
$$\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$

This line is parallel to the vector $\mathbf{b}=3\hat{\mathbf{i}}+4\hat{\mathbf{j}}+5\hat{\mathbf{k}}$

Equation of plane is $2x-2y+z=5$

Normal to the plane is $\mathbf{n}=2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$$\text{Then, } \sin\theta=\frac{\mathbf{b}\cdot\mathbf{n}}{|\mathbf{b}||\mathbf{n}|}$$
$$=\frac{|(3\hat{\mathbf{i}}+4\hat{\mathbf{j}}+5\hat{\mathbf{k}})\cdot((\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}})|}{\sqrt{3^2+4^2+5^2}\sqrt{4+4+1}}$$
$$=\frac{|6-8+5|}{\sqrt{50}\sqrt{9}}=\frac{3}{15\sqrt{2}}=\frac{1}{5\sqrt{2}}$$

Question7

The equation $xy=0$ in three-dimensional space represents

KCET 2024

Options:

A. a pair of straight lines

B. a plane

C. a pair of planes at right angles

D. a pair of parallel planes

Answer: C

Solution:

In three-dimensional space, the equation $xy = 0$ represents a set of points where the product of the coordinates x and y is zero. This can be interpreted as follows:

The product $xy = 0$ is satisfied if either $x = 0$ or $y = 0$.

The condition $x = 0$ describes the yz-plane, which is a plane that includes all points where the x-coordinate is zero.

Similarly, the condition $y = 0$ describes the xz-plane, which is a plane that includes all points where the y-coordinate is zero.

Therefore, the equation $xy = 0$ in three-dimensional space actually represents two planes that intersect along the z-axis. These two planes:

The yz-plane (where $x = 0$).

The xz-plane (where $y = 0$).

These planes are perpendicular to each other because they intersect along the z-axis at right angles. Thus, the correct answer is:

Option C: a pair of planes at right angles

Question8

The plane containing the point $(3, 2, 0)$ and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is

KCET 2024

Options:

A. $x - y + z = 1$

B. $x + y + z = 5$

C. $x + 2y - z = 1$

D. $2x - y + z = 5$

Answer: A

Solution:

Given that point $(3, 2, 0)$ lies on plane and line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ also lies on the plane.

$\because$ Plane contains points $(3, 2, 0)$ and $(3, 6, 4)$

The DR's of line joining these two points is $(0, 4, 4)$.

Therefore, DR' s of normal to plane is

$(\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \times (4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

Therefore, the equation of plane is

$4x - 4y + 4z = k$

$\because$ Point $(3, 2, 0)$ lies on planes. So, we get

$\therefore \quad k = 4 \times 3 - 4 \times 2 + 4 \times 0 = 4$

Hence, the equation of plane is $x - y + z = 1$.

Question9

If a line makes an angle of $\frac{\pi}{3}$ with each X and Y axis, then the acute angle made by Z-axis is

KCET 2023

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: C

Solution:

Given, $\alpha = \beta = \frac{\pi}{3}$

Let acute angle made by Z-axis be γ.

We know that

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2 \gamma = 1$$

$$\Rightarrow \quad \frac{1}{4} + \frac{1}{4} + \cos^2 \gamma = 1$$

$$\Rightarrow \quad \cos^2 \gamma = 1 - \frac{1}{2} \Rightarrow \cos \gamma = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \gamma = \frac{\pi}{4} \quad [\because y \text{ is acute}]$$

Question10

The length of perpendicular drawn from the point $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is

KCET 2023

Options:

A. $\sqrt{29}$

B. $\sqrt{33}$

C. $\sqrt{53}$

D. $\sqrt{66}$

Answer: C

Solution:

Let the foot point of the perpendicular drawn from the point $P(3,-1,11)$ on the straight line be L.

$\therefore$ Hence, L lies on the straight line

Let $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}=t$

$\therefore L(2t, 2+3t, 3+4t)$ [where, t is arbitrary constant]

$\therefore$ The direction ratios of PL are $(2t-3, 2+3t+1, 3+4t-11)$ or $(2t-3, 3t+3, 4t-8)$

Again, the direction ratios of the straight line.

$\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ are $(2,3,4)$.

Since, PL is perpendicular on the straight line.

Then;

$$\begin{aligned} & (2t-3)\cdot 2+(3t+3)\cdot 3+(4t-8)\cdot 4=0 \\ & 4t-6+9t+9+16t-32=0 \\ & 29t=29 \\ \because \quad & t=1 \end{aligned}$$

Hence, $L(2,5,7)$

$\therefore \quad (PL)=\sqrt{(2-3)^2+(5+1)^2+(7-11)^2}=\sqrt{53}$

Question11

The equation of the plane through the points $(2,1,0),(3,2,-2)$ and $(3,1,7)$ is

KCET 2023

Options:

A. $2x - 3y + 4z - 27 = 0$

B. $6x - 3y + 2z - 7 = 0$

C. $7x - 9y - z - 5 = 0$

D. $3x - 2y + 6z - 27 = 0$

Answer: C

Solution:

We know that the equation of a plane passing through three non-collinear points.

$(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3)

$$\because \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 2 & y - 1 & z - 0 \\ 3 - 2 & 2 - 1 & -2 - 0 \\ 3 - 2 & 1 - 1 & 7 - 0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 2 & y - 1 & z \\ 1 & 1 & -2 \\ 1 & 0 & 7 \end{vmatrix} = 0$$

$$\Rightarrow (x - 2)(7 - 0) - (y - 1)(7 + 2) + z(0 - 1) = 0$$

$$\Rightarrow 7x - 14 - (y - 1)9 + (-z) = 0$$

$$\Rightarrow 7x - 9y - z - 5 = 0$$

Question12

The point of intersection of the line $x + 1 = \frac{y+3}{3} = \frac{-z+2}{2}$ with the plane $3x + 4y + 5z = 10$ is

KCET 2023

Options:

A. $(2, -6, -4)$

B. $(2, 6, -4)$

C. $(2, 6, 4)$

D. $(-2, 6, -4)$

Answer: B

Solution:

Let $\frac{x+1}{1}=\frac{y+3}{3}=\frac{-z+2}{2}=k$

Thus, any point on this line will have co-ordinates.

$x=k-1, y=3k-3, z=-2k+2$

This line intersects the plane $3x+4y+5z=10$

Since, the value of x, y, z must satisfy equation of plane.

$$\begin{aligned} &\Rightarrow 3(k-1)+4(3k-3)+5(-2k+2)=10 \\ &\Rightarrow 3k-3+12k-12+10-10k=10 \\ &\Rightarrow \quad 5k-5=10 \\ &\Rightarrow \quad k=3 \end{aligned}$$

$\therefore$ The point of intersection is (x, y, z)

$$\begin{aligned} &\Rightarrow x=3-1, y=3\times 3-3, z=-2\times 3+2 \\ &\Rightarrow (x, y, z)=(2,6,-4) \end{aligned}$$

Question13

If $(2,3,-1)$ is the foot of the perpendicular from $(4,2,1)$ to a plane, then the equation of the plane is

KCET 2023

Options:

A. $2x+y+2z-1=0$

B. $2x-y+2z=0$

C. $2x+y+2z-5=0$

D. $2x-y+2z+1=0$

Answer: D

Solution:

Since, the line joining the two points is perpendicular to the plane, it's DR's will given the normal to the plane.

DR′s of the normal $=(4-2,2-3,1+1)=(2,-1,2)$

Hence, $a=2, b=-1$ and $c=2$

Since, the plane passes through $(2,3,-1)$.

$$\begin{aligned} &d=2(2)+3(-1)-1(2) \\ &d=-1 \end{aligned}$$

Hence, the required equation of plane is $2x-y+2z=-1$

Question14

The octant in which the point (2, -4, -7) lies is

KCET 2022

Options:

A. Eighth

B. Third

C. Fourth

D. Fifth

Answer: A

Solution:

The following table shows the signs of the coordinates in eight octants.

Octants $\rightarrow$ Coordinates $\downarrow$	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

Given, point is $(2, -4, -7)$.

It lies in VIII quadrant.

Question15

The coordinates of foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z = 29$ are

KCET 2022

Options:

A. $(2, 3, 4)$

B. $(2, -3, -4)$

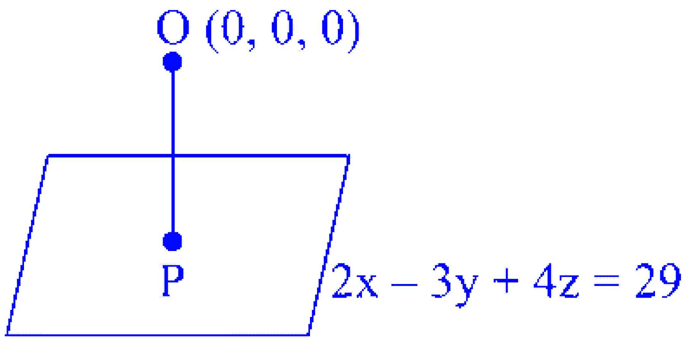
C. $(2,-3,4)$

D. $(-2,-3,4)$

Answer: C

Solution:

Given, equation of plane is $2x-3y+4z=29$



Since, OP is perpendicular to the plane

$2x-3y+4z=29$

Therefore, Direction Ratios of OP will be $<2,-3,4>$.

Equation of line OP will be

$\frac{x-0}{2}=\frac{y-0}{-3}=\frac{z-0}{4}=\lambda$ (say)

Coordinates of P will be $(2\lambda,-3\lambda,4\lambda)$.

$\because P$ lies on plane, so $2(2\lambda)-3(-3\lambda)+4(4\lambda)=29$

$\Rightarrow \quad 4\lambda+9\lambda+16\lambda=29$

$\Rightarrow \quad \lambda=1$

$\therefore$ Coordinates of P will be $(2,-3,4)$.

Question16

The angle between the pair of lines $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ and $\frac{x+1}{1}=\frac{y-4}{4}=\frac{z-5}{2}$ is

KCET 2022

Options:

A. $\theta=\cos^{-1}\left[\frac{31}{5\sqrt{42}}\right]$

B. $\theta=\cos^{-1}\left[\frac{8\sqrt{3}}{15}\right]$

C. $\theta=\cos^{-1}\left[\frac{19}{21}\right]$

D. $\theta=\cos^{-1}\left[\frac{5\sqrt{3}}{16}\right]$

Answer: A

Solution:

Given, equation of lines are

$$\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4} \quad \text{.... (i)}$$
$$\text{and} \quad \frac{x+1}{1}=\frac{y-4}{4}=\frac{z-5}{2} \quad \text{.... (ii)}$$

Direction ratios of line (i) : $<3,5,4>$

Direction ratios of line (ii) : $<1,4,2>$

We know angle between pair of lines is given by

$$\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

where, $<a_1,b_1,c_1>$ and $<a_2,b_2,c_2>$ are direction ratios of respectively lines.

$$\cos\theta=\left|\frac{3\times1+5\times4+4\times2}{\sqrt{3^2+5^2+4^4}\sqrt{1^2+4^2+2^2}}\right|$$

$$=\left|\frac{3+20+8}{\sqrt{9+25+16}\sqrt{1+16+4}}\right|=\left|\frac{31}{5\sqrt{2}\sqrt{21}}\right|=\frac{31}{5\sqrt{42}}$$

$$\theta=\cos^{-1}\left(\frac{31}{5\sqrt{42}}\right)$$

Question17

The distance of the point whose position vector is $(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}})$ from the plane $\mathbf{r}\cdot(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}})=4$ is

KCET 2022

Options:

A. $\frac{8}{\sqrt{21}}$

B. $8\sqrt{21}$

C. $-\frac{8}{\sqrt{21}}$

D. $-\frac{8}{21}$

Answer: A

Solution:

$$\begin{aligned}
&\text{Equation of plane } \mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) = 4 \\
\Rightarrow \quad & \mathbf{r} \cdot \mathbf{n} = d \\
&\text{Point } \mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}} \\
&\mathbf{n} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}, d = 4 \\
&\text{Distance } = \frac{|\mathbf{a} \cdot \mathbf{n} - d|}{|\mathbf{n}|} = \left| \frac{(2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) - 4}{\sqrt{1^2 + (-2)^2 + 4^2}} \right| \\
&= \left| \frac{2 - 2 - 4 - 4}{\sqrt{1 + 4 + 16}} \right| = \left| \frac{-8}{\sqrt{21}} \right| = \frac{8}{\sqrt{21}}
\end{aligned}$$

Question18

The equation of the line joining the points $(-3, 4, 11)$ and $(1, -2, 7)$ is

KCET 2021

Options:

A. $\frac{x+3}{2} = \frac{y-4}{3} = \frac{z-11}{4}$

B. $\frac{x+3}{-2} = \frac{y-4}{3} = \frac{z-11}{2}$

C. $\frac{x+3}{-2} = \frac{y+4}{3} = \frac{z+11}{4}$

D. $\frac{x+3}{2} = \frac{y+4}{-3} = \frac{z+11}{2}$

Answer: B

Solution:

Given, points are $(-3, 4, 11)$ and $(1, -2, 7)$.

Let $A(-3, 4, 11)$ and $B(1, -2, 7)$

Direction ratios of $AB = 1 - (-3), -2 - 4, 7 - 11$

$$\begin{aligned}
&= 4, -6, -4 \\
&= -23, 2
\end{aligned}$$

Now, we have one point $(-3, 4, 11) = (x_1, y_1, z_1)$ and direction ratios $(a, b, c) = -2, 3, 2$

$\because$ Equation of line :

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$
$$\Rightarrow \quad \frac{x+3}{-2}=\frac{y-4}{3}=\frac{z-11}{2}$$

Question19

The angle between the lines whose direction cosines are $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{-\sqrt{3}}{2}\right)$

KCET 2021

Options:

A. π

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

Given, direction cosines of line 1 : (l_1, m_1, n_1)

$=\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$

direction cosines of line 2 $\Rightarrow (l_2, m_2, n_2)$

$=\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2}\right)$

We know that, angle between two lines is given by $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

$$\begin{aligned}\cos\theta &= \left|\frac{\sqrt{3}}{4}\times\frac{\sqrt{3}}{4}+\frac{1}{4}\times\frac{1}{4}-\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}\right| \\ &= \left|\frac{3}{16}+\frac{1}{16}-\frac{3}{4}\right| = \left|\frac{3+1-12}{16}\right| \\ &= \left|-\frac{8}{16}\right| = \frac{1}{2}\end{aligned}$$
$$\therefore \quad \theta = \frac{\pi}{3}$$

Question20

If a plane meets the coordinate axes at A, B and C in such a way that the centroid of $\triangle ABC$ is at the point $(1,2,3)$, then the equation of the plane is

KCET 2021

Options:

A. $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$

B. $\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$

C. $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=\frac{1}{3}$

D. $\frac{x}{1}-\frac{y}{2}+\frac{z}{3}=-1$

Answer: B

Solution:

Given, plane meets the coordinate axes at A, B and C.

Let the plane meets the coordinate axes at the points $A(a,0,0), B(0,b,0), C(0,0,c)$

The equation of plane is

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ .... (i)

Centroid formula says,

$$x=\frac{x_1+x_2+x_3}{3}, y=\frac{y_1+y_2+y_3}{3},$$
$$z=\frac{z_1+z_2+z_3}{3}$$

Centroid of $\triangle ABC=\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

Given centroid $=(1,2,3)$

$\Rightarrow\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

Therefore, $a=3, b=6$ and $c=9$

Put the value of a, b and c in equation of plane (i), we get

$\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$

Question21

The area of the quadrilateral $ABCD$ when $A(0,4,1), B(2,3,-1), C(4,5,0)$ and $D(2,6,2)$ is equal to

KCET 2021

Options:

A. 9 sq units

B. 18 sq units

C. 27 sq units

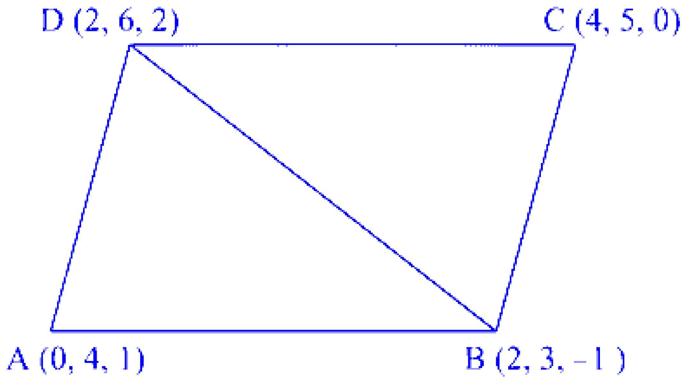
D. 81 sq units

Answer: A

Solution:

In quadrilateral $ABCD$, given that $A(0,4,1), B(2,3,-1), C(4,5,0)$ and $D(2,6,2)$.

$$\mathbf{AB} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$
$$\mathbf{AD} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$$
$$\mathbf{CB} = -2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$$
$$\mathbf{CD} = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$



Required area of quadrilateral

$$= \frac{1}{2}|\mathbf{AB} \times \mathbf{AD}| + \frac{1}{2}|\mathbf{CB} \times \mathbf{CD}|$$
$$= \frac{1}{2}|3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}}| + \frac{1}{2}| - 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 6\hat{\mathbf{k}}|$$
$$= \frac{1}{2}\sqrt{3^2 + (-6)^2 + 6^2} + \frac{1}{2}\sqrt{(-3)^2 + 6^2 + (-6)^2}$$
$$= \frac{1}{2} \times 9 + \frac{1}{2} \times 9 = 9 \text{ sq units}$$

Question22

The mid points of the sides of triangle are $(1,5,-1)(0,4,-2)$ and $(2,3,4)$ then centroid of the triangle

KCET 2021

Options:

A. $(1,4,3)$

B. $\left(1,4,\frac{1}{3}\right)$

C. $(-1,4,3)$

D. $\left(\frac{1}{3},2,4\right)$

Answer: B

Solution:

Given, mid points of the sides of triangle are $(1,5,-1),(0,4,-2)$ and $(2,3,4)$.

Let,

$(x_1,y_1,z_1)=(1,5,-1),$
$(x_2,y_2,z_2)=(0,4,-2),$
$(x_3,y_3,z_3)=(2,3,4).$

Then centroid of triangle, $G(x,y,z)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$

$$G(x,y,z)=\left(\frac{1+0+2}{3},\frac{5+4+3}{3},\frac{-1-2+4}{3}\right)$$
$$=\left(1,4,\frac{1}{3}\right)$$

Question23

The point $(1,-3,4)$ lies in the octant

KCET 2020

Options:

A. Second

B. Third

C. Fourth

D. Eighth

Answer: C

Solution:

Clearly, point $(1,-3,4)$ lies in IV octant.

Question24

The distance of the point $(1,2,-4)$ from the line $\frac{x-3}{2}=\frac{y-3}{3}=\frac{z+5}{6}$ is

KCET 2020

Options:

A. $\frac{293}{7}$

B. $\frac{\sqrt{293}}{7}$

C. $\frac{293}{49}$

D. $\frac{\sqrt{293}}{49}$

Answer: B

Solution:

Given, point $(1,2,-4)$.

and line $\frac{x-3}{2}=\frac{y-3}{3}=\frac{z+5}{6}=\lambda$ (let)

$\therefore \quad x=2\lambda+3$
$\therefore \quad x=2\lambda+3, y=3\lambda+3, z=6\lambda-5$

Let $P(2\lambda+3,3\lambda+3,6\lambda-5)$ be the foot of perpendicular from $A(1,2,4)$

$\therefore$ Direction ratios of AP are

$2\lambda+3-1,3\lambda+3-2,6\lambda-5+4$
i.e., $2\lambda+2,3\lambda+1,6\lambda-1$

And direction ratios of given line are $2,3,6$: Since, $AP \perp$ given line

$\therefore 2(2\lambda+2)+3(3\lambda+1)+6(6\lambda-1)=0$
$\Rightarrow \quad 4\lambda+4+9\lambda+3+36\lambda-6=0$
$\Rightarrow \quad 49\lambda+1=0$
$\Rightarrow$
$\lambda=-\dfrac{1}{49}$
$\therefore P(2\lambda+3,3\lambda+3,6\lambda-5)$
$=4\lambda^2+4+8\lambda+9\lambda^2+1+6\lambda+36\lambda^2+1-12\lambda$
$=49\lambda^2+6+2\lambda$

$\therefore$ Distance of the point $(1,2,-4)$ from the given line,

line,

$$AP^2 = (2\lambda+2)^2 + (3\lambda+1)^2 + (6\lambda-1)^2$$
$$= 4\lambda^2 + 4 + 8\lambda + 9\lambda^2 + 1 + 6\lambda + 3$$
$$= 49\lambda^2 + 6 + 2\lambda$$
$$= 49\left(-\frac{1}{49}\right)^2 + 6 + 2\left(-\frac{1}{49}\right)$$
$$= \frac{1}{49} + 6 - \frac{2}{49}$$
$$= 6 - \frac{1}{49} = \frac{293}{49}$$
$$\Rightarrow AP = \frac{\sqrt{293}}{7}$$

Question25

The sine of the angle between the straight line $\frac{x-2}{3} = \frac{3-y}{-4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

KCET 2020

Options:

A. $\frac{3}{\sqrt{30}}$

B. $\frac{3}{50}$

C. $\frac{4}{5\sqrt{2}}$

D. $\frac{\sqrt{2}}{10}$

Answer: D

Solution:

Given straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and plane $2x - 2y + z = 5$

Let θ be the angle between line and plane then

$$\sin\theta = \left|\frac{3\times 2 + (4)(-2) + 5\times 1}{\sqrt{(3)^2+(4)^2+(5)^2}\sqrt{(2)^2+(-2)^2+(1)^2}}\right|$$
$$= \left|\frac{6-8+5}{\sqrt{9+16+25}\sqrt{4+4+1}}\right|$$
$$= \left|\frac{3}{\sqrt{50}\sqrt{9}}\right| = \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$$

Question26

If a line makes an angle of with each of X and Y-axis, then the acute angle made by Z-axis is

KCET 2020

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: A

Solution:

Given, $l = m = \cos\frac{\pi}{3} = \frac{1}{2}$

$\therefore \quad l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + n^2 = 1$

$\Rightarrow \quad \frac{1}{4} + \frac{1}{4} + n^2 = 1$

$\Rightarrow \quad n^2 = \frac{1}{2} \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

Then the acute angle made by Z-axis is $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

Question27

Foot of the perpendicular drawn from the point $(1, 3, 4)$ to the plane $2x - y + z + 3 = 0$ is

KCET 2019

Options:

A. $(1, 2 - 3)$

B. $(-1, 4, 3)$

C. $(-3, 5, 2)$

D. $(0, -4, -7)$

Answer: B

Solution:

Equation of perpendicular line (normal to the given plane) passing through the point $(1, 3, 4)$ is

$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda$ (let) .... (i)

Co-ordinates of any point on line is $(2\lambda + 1, -\lambda + 3, \lambda + 4)$ which is also lies

On the given plane $2x - y + z + 3 = 0$

$$\therefore 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$$
$$\Rightarrow \quad 4\lambda + 2 + \lambda - 3 + \lambda + 4 + 3 = 0$$
$$\Rightarrow \quad 6\lambda + 6 = 0 \Rightarrow \lambda = -1$$

$\therefore$ required foot of perpendicular $(2\lambda + 1, -\lambda + 3, \lambda + 4)$ i.e. $(-2 + 1, 1 + 3, -1 + 4) = (-1, 4, 3)$

Question28

Acute angel between the line $\frac{(x-5)}{2} = \frac{y+1}{-1} = \frac{z+4}{1}$ and the plane $3x - 4y - z + 5 = 0$ is

KCET 2019

Options:

A. $\cos^{-1}\left(\frac{5}{2\sqrt{3}}\right)$

B. $\cos^{-1}\left(\frac{9}{\sqrt{364}}\right)$

C. $\sin^{-1}\left(\frac{5}{2\sqrt{13}}\right)$

D. $\sin^{-1}\left(\frac{9}{\sqrt{364}}\right)$

Answer: A

Solution:

Key Idea the angle θ between line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and plane $a_2x + b_2 + c_2 = d$ is given by

$\sin\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

$\therefore$ required acute angle between given line $\frac{x-5}{2}=\frac{y+1}{-1}=\frac{z+4}{1}$ and plane $3x-4y-z+5=0$ is given by

$$\theta=\sin^{-1}\left(\frac{|(2(3)+(-1)(-4)+(1)(-1)}{\sqrt{2^2+(-1)^2+(1)^2}\sqrt{(3)^2+(-4)^2+(-1)^2}}\right)$$
$$=\sin^{-1}\left(\frac{9}{\sqrt{6}\sqrt{26}}\right)=\cos^{-1}\left(\frac{5}{\sqrt{2}\sqrt{26}}\right)$$

thus, any option is not matching

Question29

The distance of the point $(1,2,1)$ from the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$ is.

KCET 2019

Options:

A. $\frac{\sqrt{5}}{3}$

B. $\frac{2\sqrt{3}}{5}$

C. $\frac{20}{3}$

D. $\frac{2\sqrt{5}}{3}$

Answer: D

Solution:

Given point $(1,2,1)$ and line

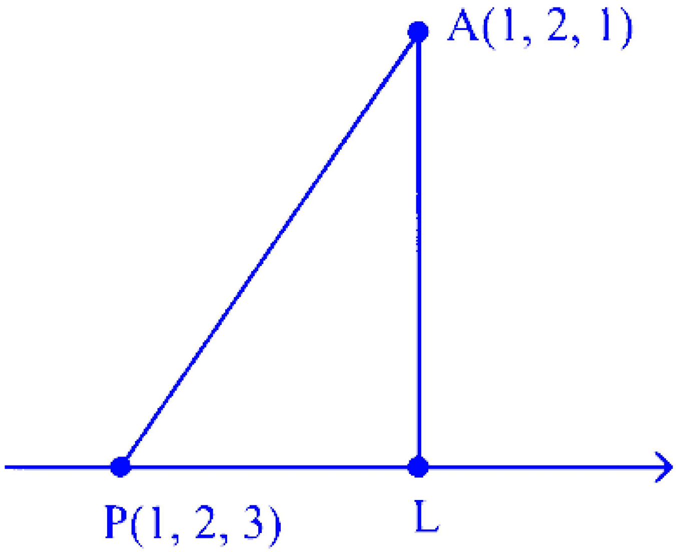
$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$

Here, $\mathbf{PQ}=2\hat{\mathbf{i}}+\mathbf{j}+2\hat{\mathbf{k}}$

and $\mathbf{PA}=0\hat{\mathbf{i}}+0\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

$\therefore$ required distance

$AL=\frac{|\mathbf{PQ}\times\mathbf{PA}|}{|\mathbf{PQ}|}$ .... (i)



Now, $\mathbf{PQ} \times \mathbf{PA} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & 2 \\ 0 & 0 & -2 \end{vmatrix}$

$= \hat{\mathbf{i}}(-2-0) - \hat{\mathbf{j}}(-4-0) + \hat{\mathbf{k}}(0-0)$

$= -2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$\therefore |\mathbf{PQ} \times \mathbf{PA}| = \sqrt{(-2)^2 + (4)^2} = \sqrt{20}$

and $|PQ| = \sqrt{(2)^2 + (1)^2 + (2)^2} = 3$

$\therefore \quad AL = \frac{\sqrt{20}}{3} = \frac{2\sqrt{5}}{3}$ [from (i)]

Question30

XY plane divides the line joining the points $A(2, 3, -5)$ and $B(-1, -2, -3)$ in the ratio

KCET 2019

Options:

A. 5 : 3 internally

B. 2 : 1 internally

C. 5 : 3 externally

D. 3 : 2 externally

Answer: C

Solution:

The equation of the line joining the point $A(2, 3, -5)$ and $B(-1, -2, -3)$ is

$$\frac{x-2}{-1-2}=\frac{y-3}{-2-3}=\frac{z+5}{-3+5}$$
$$\Rightarrow \quad \frac{x-2}{-3}=\frac{y-3}{-5}=\frac{z+5}{2}=\lambda(\text{ say })$$

Any point on this line is $(-3\lambda+2-5\lambda+3, 2\lambda-5)$

Since, XY-plane divides the line joining the points A and $B \therefore z=0$

$\Rightarrow \quad 2\lambda-5=0 \Rightarrow \lambda=\frac{5}{2}$

$\therefore$ any point on XY-plane which divides the line joining the points A and B.

$\left(-3\times\frac{5}{2}+2, -5\times\frac{5}{2}+3, 0\right)$ i.e. $\left(\frac{-11}{2}, \frac{-19}{2}, 0\right)$

Now, by using section formula.

$\frac{mx(-1)+nx(2)}{m+n}=\frac{-11}{2} \Rightarrow \frac{m}{n}=\frac{-15}{9}=\frac{-5}{3}$

$\Rightarrow XY$-plane divides externally in ratio $5:3$ the line join points

----

Question31

The image of the point $(1,6,3)$ in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ is

KCET 2018

Options:

A. $(1,0,7)$

B. $(7,0,1)$

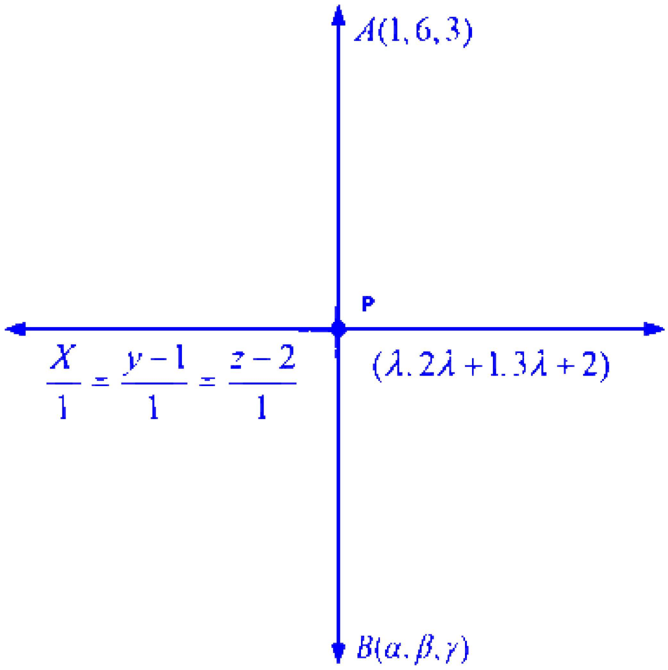
C. $(2,7,0)$

D. $(-1,-6,-3)$

Answer: A

Solution:

Let any point P lie on the line



$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is $P(\lambda, 2\lambda+1, 3\lambda+2)$

Direction ratio of line PA is $(\lambda-1, 2\lambda-5, 3\lambda-1)$

Since, line PA is perpendicular to the given line.

$\therefore \quad 1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$

$\Rightarrow \quad \lambda-1+4\lambda-10+9\lambda-3=0$

$\Rightarrow \quad 14\lambda=14 \Rightarrow \lambda=1$

point $P(1,3,5)$

P is the mid-point of AB

$$\therefore \quad 1=\frac{\alpha+1}{2} \Rightarrow \alpha=1$$
$$3=\frac{\beta+6}{2} \Rightarrow \beta=0$$
$$5=\frac{\gamma+3}{2} \Rightarrow \gamma=7$$

$\therefore$ Image of the point $(1,6,3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is $(1,0,7)$.

Question32

The angle between the lines $2x=3y=-z$ and $6x=-y=-4z$ is

KCET 2018

Options:

A. 0°

B. 45°

C. 90°

D. 30°

Answer: C

Solution:

Given lines are:

$2x = 3y = -z$

$6x = -y = -4z$

From these equations, we can express the lines in parametric form:

For the first line:

$\frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{-1}$

This gives the direction ratios (or direction vector) for the first line as $a_1 = \frac{1}{2}$, $b_1 = \frac{1}{3}$, and $c_1 = -1$.

For the second line:

$\frac{x}{\frac{1}{6}} = \frac{y}{-1} = \frac{z}{-\frac{1}{4}}$

This gives the direction ratios (or direction vector) for the second line as $a_2 = \frac{1}{6}$, $b_2 = -1$, and $c_2 = -\frac{1}{4}$.

To find the angle θ between the two lines, we use the formula for the cosine of the angle between two vectors:

$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{(a_1^2+b_1^2+c_1^2)(a_2^2+b_2^2+c_2^2)}}$

Substitute the values:

$\cos\theta = \frac{\frac{1}{2}\times\frac{1}{6}+\frac{1}{3}\times(-1)+(-1)\times\left(-\frac{1}{4}\right)}{\sqrt{\left(\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2+(-1)^2\right)\left(\left(\frac{1}{6}\right)^2+(-1)^2+\left(-\frac{1}{4}\right)^2\right)}}$

Simplify the components:

Numerator:

$\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{3}\cdot(-1)+(-1)\cdot\left(-\frac{1}{4}\right) = \frac{1}{12}-\frac{1}{3}+\frac{1}{4}$

Denominator:

$\sqrt{\left(\frac{1}{4}+\frac{1}{9}+1\right)\left(\frac{1}{36}+1+\frac{1}{16}\right)}$

Calculate each term:

Numerator:

$\frac{1}{12}-\frac{1}{3}+\frac{1}{4} = \frac{1}{12}-\frac{4}{12}+\frac{3}{12} = 0$

Since the numerator is zero, $\cos\theta = 0$.

Thus, $\theta = 90°$.

Therefore, the angle between the lines is 90°.

Question33

The value of k such that the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies on the plane $2x - 4y + z = 7$ is

KCET 2018

Options:

A. -7

B. 4

C. -4

D. 7

Answer: D

Solution:

To find the value of k such that the line

$\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$

lies on the plane

$2x - 4y + z = 7$

we begin by noting the point $(4, 2, k)$ from the line should satisfy the plane equation. Plugging this point into the plane equation, we have:

$2(4) - 4(2) + k = 7$

Simplifying the equation:

$8 - 8 + k = 7$

which simplifies further to:

$k = 7$

Thus, the value of k is 7.

Question34

The locus represented by $xy + yz = 0$ is

KCET 2018

Options:

A. a pair of perpendicular lines

B. a pair of parallel lines

C. a pair of parallel planes

D. a pair of perpendicular planes

Answer: D

Solution:

We start with the equation:

$xy + yz = 0.$

Step 1. Factor the equation by taking y as a common factor:

$y(x + z) = 0.$

Step 2. Apply the zero-product property to get:

Either $y = 0$, or

$x + z = 0.$

These represent two distinct planes in three-dimensional space:

The plane $\Pi_1 : y = 0$ (the xz-plane).

The plane $\Pi_2 : x + z = 0.$

Step 3. Check if these two planes are perpendicular by determining their normal vectors.

For the plane $\Pi_1 : y = 0$, the normal vector is:

$\mathbf{n}_1 = (0, 1, 0).$

For the plane $\Pi_2 : x + z = 0$, the normal vector is:

$\mathbf{n}_2 = (1, 0, 1).$

Step 4. Compute the dot product of the normal vectors:

$\mathbf{n}_1 \cdot \mathbf{n}_2 = (0)(1) + (1)(0) + (0)(1) = 0.$

Since the dot product is zero, the normal vectors are perpendicular, which implies that the planes are also perpendicular.

Conclusion: The locus represented by $xy + yz = 0$ is a pair of perpendicular planes.

Therefore, the correct option is:

Option D (a pair of perpendicular planes).

Question35

The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with X-axis, the value of α is equal to

KCET 2017

Options:

A. $\frac{\sqrt{2}}{3}$

B. $\frac{2}{7}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{3}{7}$

Answer: B

Solution:

To determine the angle that the given plane makes with the X-axis, we start with the equation of the plane:

$2x - 3y + 6z - 11 = 0$

The direction ratios (dr') of the normal to the plane are $\langle 2, -3, 6 \rangle$. The direction ratios of the X-axis are $\langle 1, 0, 0 \rangle$.

The angle θ between the plane and the X-axis can be found using the formula for the sine of the angle between two vectors:

$\sin\theta = \frac{\text{dot product of the normal vector and the X-axis vector}}{\|\text{normal vector}\| \cdot \|\text{X-axis vector}\|}$

Calculating the dot product:

$2 \times 1 + (-3) \times 0 + 6 \times 0 = 2$

Calculating the magnitude of the normal vector:

$\sqrt{2^2 + (-3)^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

The magnitude of the X-axis vector:

$\sqrt{1^2 + 0^2 + 0^2} = 1$

Thus, the sine of the angle is:

$\sin\theta = \frac{2}{7 \times 1} = \frac{2}{7}$

Therefore, the angle θ is given by:

$\theta = \sin^{-1}\left(\frac{2}{7}\right)$

Comparing this with the given angle $\sin^{-1}(\alpha)$, we find that $\alpha = \frac{2}{7}$.

Question36

The perpendicular distance of the point $P(6, 7, 8)$ from XY-plane is

KCET 2017

Options:

A. 6

B. 7

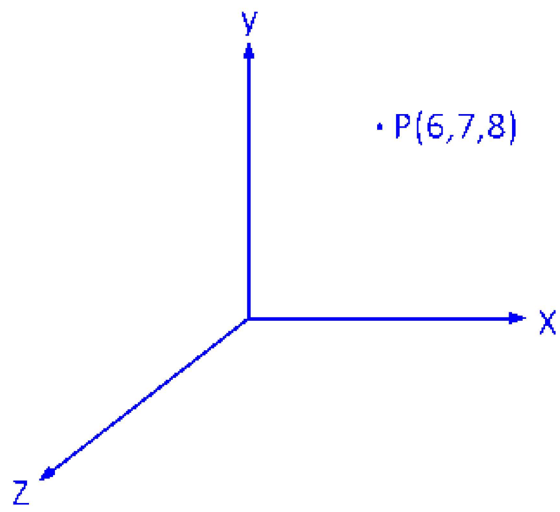
C. 5

D. 8

Answer: D

Solution:

According to the question,



Distance of any point from XY - plane is $= |z| = |8| = 8$

Question37

Reflexion of the point (α, β, γ) in XY-plane is

KCET 2017

Options:

A. $(0, 0, \gamma)$

B. $(-\alpha, -\beta, \gamma)$

C. $(\alpha, \beta, -\gamma)$

D. $(\alpha, \beta, 0)$

Answer: C

Solution:

The reflection of any point (x, y, z) in the XY-plane results in the point $(x, y, -z)$.

Therefore, the reflection of the point (α, β, γ) in the XY-plane is $(\alpha, \beta, -\gamma)$.

Question38

The distance of the point $(-2, 4, -5)$ from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is

KCET 2017

Options:

A. $\sqrt{\frac{37}{10}}$

B. $\frac{37}{10}$

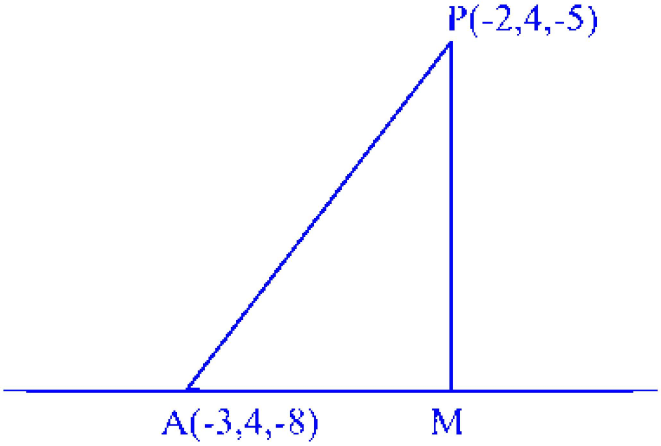
C. $\frac{\sqrt{37}}{10}$

D. $\frac{37}{\sqrt{10}}$

Answer: A

Solution:

The line passes through $A(-3,4,-8)$ and is parallel to the vector $\mathbf{b}=3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+6\hat{\mathbf{k}}$



Let M be the foot of the perpendicular from $P(-2,4,-5)$ on the given line.

We, have

$\mathbf{AP}=-\hat{\mathbf{i}}+0\hat{\mathbf{j}}-3\hat{\mathbf{k}}$

$$\Rightarrow \quad |\mathbf{AP}|=\sqrt{1+0+9}=\sqrt{10}$$

Clearly, $AM=$ Projection of $\mathbf{AP}$ on $\mathbf{b}$

$$\begin{aligned}
\Rightarrow \quad AM &= \left|\frac{\mathbf{AP}\cdot\mathbf{b}}{\mathbf{b}}\right| \\
&= \left|\frac{(-\hat{\mathbf{i}}-3\hat{\mathbf{k}})\cdot(3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+6\hat{\mathbf{k}})}{|3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+6\hat{\mathbf{k}}|}\right| \\
&= \left|\frac{-3-18}{\sqrt{9+25+36}}\right| = \left|\frac{-21}{\sqrt{70}}\right| = \frac{21}{\sqrt{70}} \\
\therefore \quad PM &= \sqrt{AP^2-AM^2} \\
&= \sqrt{10-\frac{441}{70}} = \sqrt{\frac{259}{70}} \\
&= \sqrt{\frac{37}{10}}
\end{aligned}$$
